Small-time Asymptotics of Call Prices and Implied Volatilities for Exponential Lévy Models

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We derive call-price and implied volatility asymptotic expansions in time to maturity for a selection of exponential Lévy models. We consider asset-price models whose log returns structure is a Lévy process. In particular, we consider Lévy processes of the form $(L_t + \sigma W_t)_{t\geq 0}$ where $L = (L_t)_{t\geq 0}$ is a pure-jump Lévy process in the domain of attraction of a stable random variable, $W = (W_t)_{t\geq 0}$ is a standard Brownian motion independent of L, and $\sigma \geq 0$.

Call-price asymptotics for in-the-money (ITM) and out-of-the-money (OTM) options are extensively covered in the literature; however, at-the-money (ATM) call-price asymptotics under exponential Lévy models are relatively new.

In this thesis, we consider two main problems. First, we consider very general Lévy models for L. More specifically, L that are in the domain of attraction of a stable random variable. Under some relatively minor assumptions, we give first-order call-price and implied volatility asymptotics.

Interestingly, in the case where $\sigma = 0$ new orders of convergence are discovered which show a much richer structure than was previously considered. Concretely, we show that the rate of convergence can be of the form $t^{1/\alpha}\ell(t)$ where ℓ is a slowly varying function. We also give an example of a Lévy model which exhibits this new type of behavior and has a new order of convergence where ℓ is not asymptotically constant.

In the case where $\sigma \neq 0$, we show that the Brownian component is the dominant term in the asymptotic expansion of the call-price. Under more general conditions on L (even removing the requirement of L to be in the domain of attraction of a stable

random variable), we show that the first-order call-price asymptotics are of the order $\sqrt{t}$.

Finally, we investigate the CGMY process. For this process, call-price asymptotics are known to third order. Previously, measure transformation and technical estimation methods were the only tools available for proving the order of convergence. In the last chapter, we give a new method that relies on the Lipton-Lewis (LL) formula. Using the LL formula guarantees that we can estimate the call-price asymptotics using only the characteristic function of the Lévy process. While this method does not provide a less technical approach, it is novel and is promising for obtaining second-order call-price asymptotics for ATM options for a more general class of Lévy processes.