MINIMUM-FUEL POWERED DESCENT IN THE PRESENCE OF RANDOM DISTURBANCES

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Abstract

It has recently been shown that minimum-fuel powered descent guidance can be solved onboard as a convex optimization problem. It therefore presents itself as a promising technology to enable future planetary exploration missions. However, since this approach is formulated as a deterministic optimal control problem, the resulting guidance law is only designed for a single pair of initial and target states without external disturbances.

We attempt to extend this approach to the more general case of steering initial position and velocity distributions to target distributions, while considering Brownian motion process noise acting on the system.

System Model

Powered descent with random external force

$$\mathrm{d}\dot{r} = (u/m + g)\mathrm{d}t + (\gamma/m)\mathrm{d}w$$

Assume control structure

$$u = \bar{u} + K\tilde{x} \qquad \mathrm{E}\,\|u\| = \mathrm{E}\,\|\bar{u} + \tilde{u}\| \approx \|\bar{u}\|$$

Mass change is given by

$$\dot{m} = -\alpha\,\|\bar{u}\|$$

The mean trajectory therefore satisfies

$$\ddot{\bar{r}} = \bar{u}/m + g$$

The disturbance is given by the stochastic system

$$\mathrm{d}\dot{\tilde{r}} = (\tilde{u}/m)\mathrm{d}t + (\gamma/m)\mathrm{d}w$$

with state covariance subject to

$$\dot{P}_x = (A + B_m K)P_x + P_x(A + B_m K)^\intercal + \gamma^2 B_m B_m^\intercal$$

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ I/m(t) \end{bmatrix}$$

and control covariance

$$P_u = \mathrm{E}[\tilde{u}\tilde{u}^\intercal] = K P_x K^\intercal$$

In summary, by assuming mean control is much larger than feedback component we separate mean and disturbance into separate but interdependent systems.

Closed-Loop Control Variance

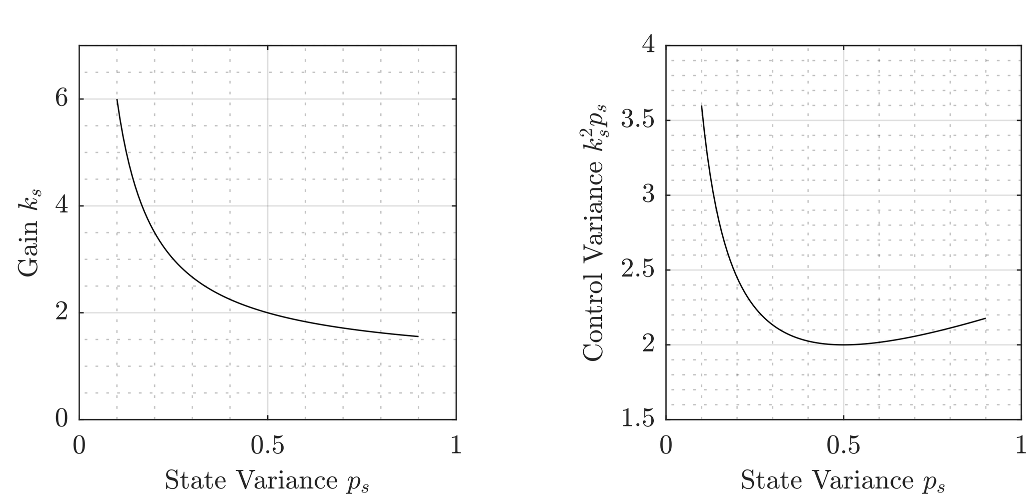
Consider the scalar stochastic system

$$\mathrm{d}x = (a - bk)x\mathrm{d}t + \mathrm{d}w, \qquad \dot{p} = 2(a - bk)p + 1$$

As gain increases to infinity, decrease variance to zero. The control is also a random variable

$$u = -kx$$

But we can minimize control variance $k^2 p$



Left: Gain required to maintain steady-state state variance. Right: Steady-state control variance plotted against steady-state state variance.

Problem Formulation

We want to design a control pair $(\bar{u}, K)$ to achieve soft landing at final time $t_f > 0$

$$\bar{x}(t_f) = 0, \quad P_x(t_f) = P_{x_f},$$

Control is constrained to the set

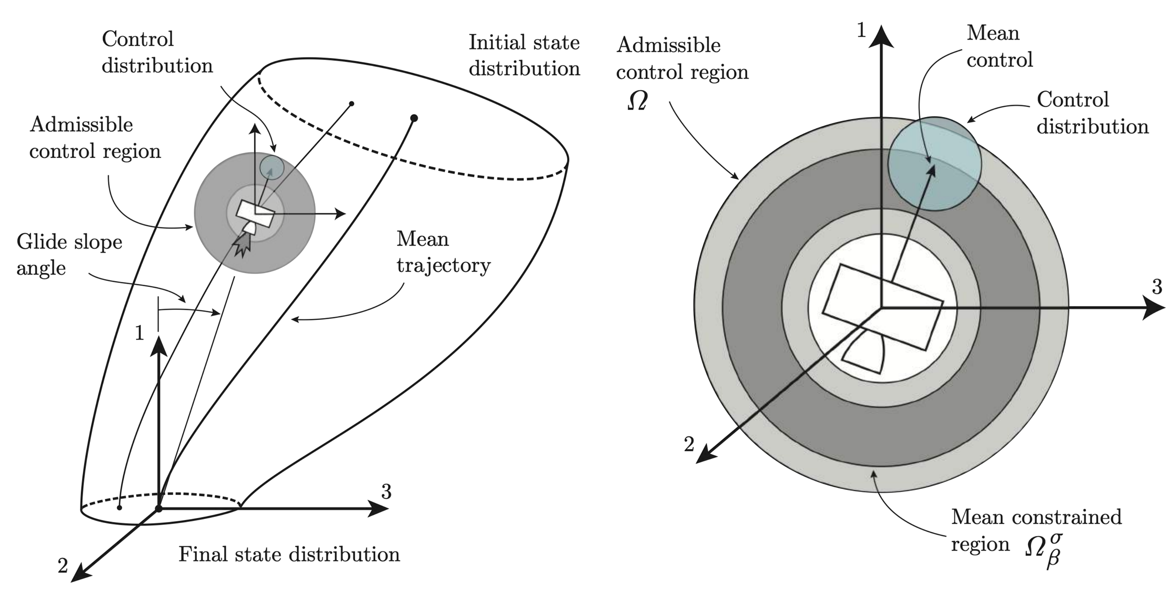
$$\Omega = \{z \in R^3 : \rho_1 \leq \|z\| \leq \rho_2\}$$

Enforce constraint in probability (Gaussian dist.)

$$\Pr[u \in \Omega] = \int_\Omega f(z, \bar{u}, P_u)\mathrm{d}z \geq 1 - \beta$$

Minimize mean fuel cost

$$J(\bar{u}) = \int_0^{t_f} \|\bar{u}(t)\|\,\mathrm{d}t$$

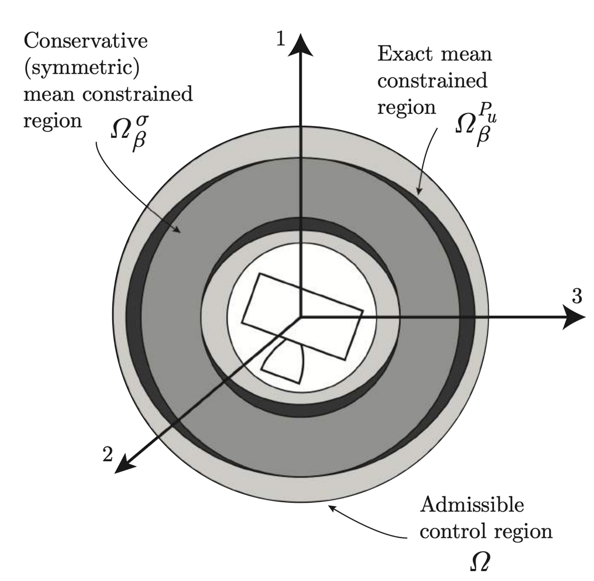


Thrust Constraint in Probability

Probability constraint is equiv. to constraining the mean control vector to the set

$$\Omega_\beta^{P_u} = \{\bar{u} \in \Omega : \int_\Omega f(z, \bar{u}, P_u)\mathrm{d}z \geq 1 - \beta\}.$$

Relax with max singular value $\sigma_u^2 = \sigma_{\max}^2(P_u)$



Using radial symmetry,

$$\rho_1^\sigma = \min\{\|\bar{u}\| : \bar{u} \in \Omega_\beta^\sigma\},$$

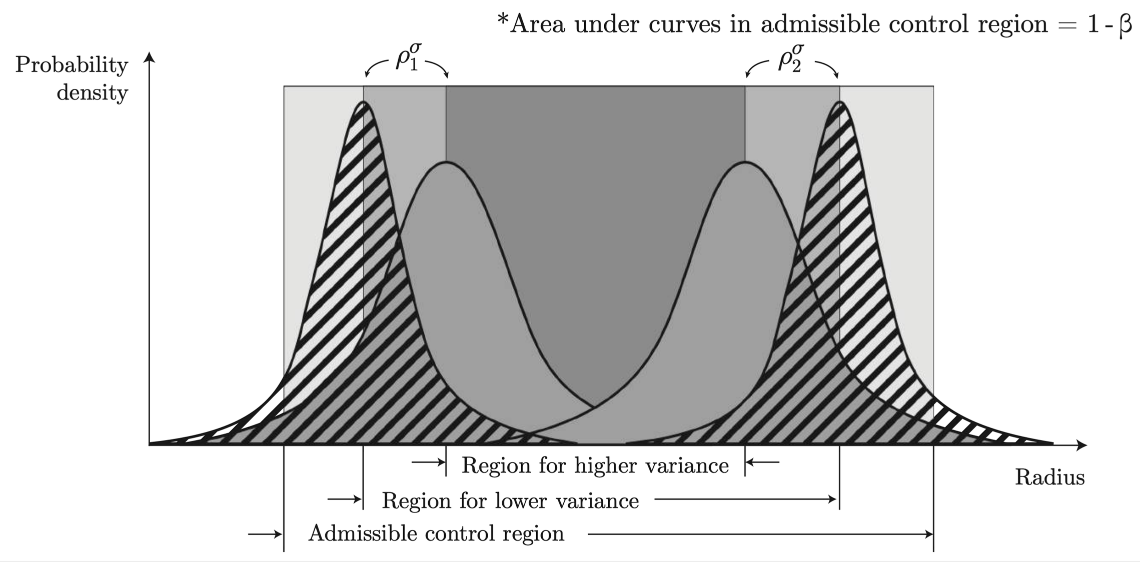
$$\rho_2^\sigma = \max\{\|\bar{u}\| : \bar{u} \in \Omega_\beta^\sigma\}.$$

Then if

$$\rho_1^\sigma(t) \leq \|\bar{u}(t)\| \leq \rho_2^\sigma(t)$$

the probability constraint is satisfied

*Area under curves in admissible control region = 1 - β



Mean and Covariance Steering

- We separated the mean and disturbance into two interdependent systems
  - Covariance depends on mass
  - Mean thrust bound depends on closed-loop control covariance
- Solve mean steering as a convex program
- For given mass profile, there is a closed form solution to covariance steering problem:

$$\begin{aligned} \min_K \quad & \int_0^{t_f} \mathrm{tr}\,Q_u P_u + \mathrm{tr}\,Q_x P_x\,\mathrm{d}t \\ \text{s.t.} \quad & \dot{P}_x = (A + B_m K)P_x + P_x(A + B_m K)^\intercal + \gamma^2 B_m B_m^\intercal \\ & P_u = K P_x K^\intercal \\ & P_x(0) = P_{x_0}, \quad P_x(t_f) = P_{x_f} \end{aligned}$$

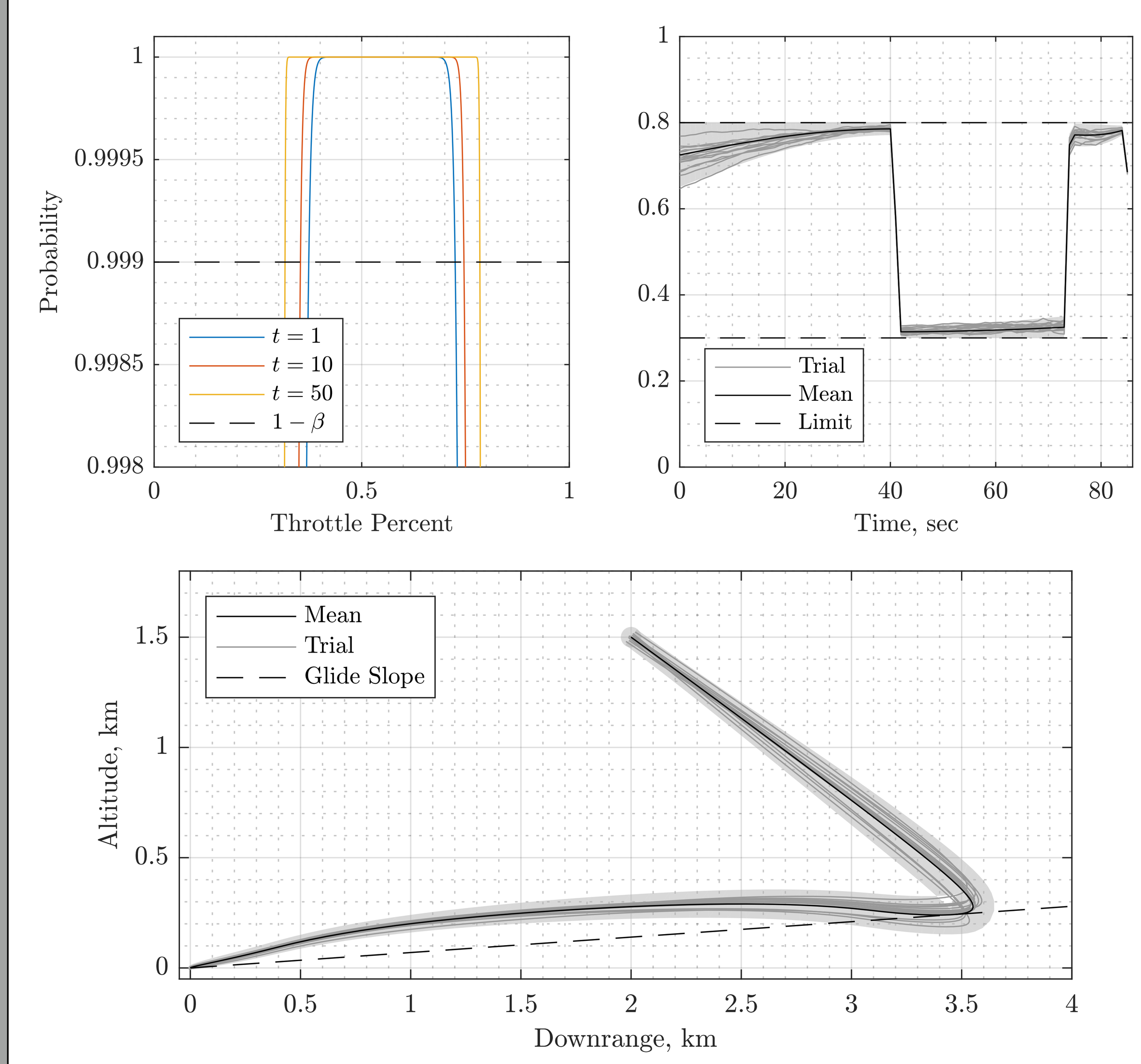
Simulation Results

MSL divert scenario: 1,500 m altitude, 125 m/s velocity at flight path angle -36.9 deg. Command divert to site 2,000 m behind vehicle in plane of velocity vector.

$$P_{x_0} = \mathrm{diag}(200, 200, 200, 10, 10, 10)$$
$$P_{x_f} = \mathrm{diag}(10, 10, 10, 1, 1, 1)$$

Enforce that control is in bounds with probability 99.9%. Settings summarized below:

Property	Value	Unit
Wet mass m_0	1,905	kg
Propellant mass	400	kg
α	4.4865e-04	kg/N sec
P_{x_0}	diag(200, 200, 200, 10, 10, 10)	m^2/s^2, m^2
P_{x_f}	diag(10, 10, 10, 1, 1, 1)	m^2/s^2, m^2
$\bar{r}_0$	(1500, 0, 2000)	m
$\dot{\bar{r}}_0$	(−75, 0, 100)	m/s



Left: probability that u is in Ω is plotted against the mean throttle percent at different times in the simulation. The values of ρ_1^σ and ρ_2^σ are determined by the left and right intersections of the probability curve with the dashed line for $1 - \beta$ where $\beta = 0.001$. Right: mean throttle percent and throttle histories from select Monte Carlo trials. The shaded region contains 99.9% of throttle histories. Bottom: Monte Carlo trials with 99.9% of trajectories in the shaded region.

Conclusions / Future Work

Conclusions

- Presented a stochastic extension to optimal powered descent that guarantees throttle constraints in probability
- Constraints on feedback control introduced a coupling between trajectory and control
- This work constrained that the control would not saturate, but a better constraint would be on the final state covariance (allowing saturation)

Future Work

- May be possible to generalize theory to any minimum-fuel optimal control problem where there is feedback
- Handle parametric uncertainty
- Study possible application to entry guidance in an uncertain atmosphere

References

[1] Y. Chen, T. T. Georgiou, and M. Pavon (2016) IEEE Trans. on Automatic Control 61, 1158–1169.
[2] J. Ridderhof and P. Tsiotras (2018) AIAA SciTech Forum.

Acknowledgements

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